Kinetic treatment for the exact solution of the unsteady Rayleigh flow problem of a rarefied homogeneous charged gas bounded by an oscillating plate

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Abstract: The exact solution of the unsteady Rayleigh flow problem of a rarefied charged gas bounded by an oscillating plate has been made. For this purpose, we use the traveling wave solution method. The kinetic and the irreversible thermodynamic properties of the charged gas are presented from the molecular viewpoint. Our study is based on the solution of the Bhatnager–Gross–Krook (BGK) model of the Boltzmann kinetic equation, with the precision value of the electron–electron collision frequency. The BGK model equation coupled with Maxwell’s equations, for electron gas near an oscillating rigid plane, are solved. The distinction and comparisons between the perturbed and the equilibrium velocity distribution functions are illustrated. The ratios between the different contributions of the internal energy changes are predicted via the extended Gibbs equation for both diamagnetic and paramagnetic plasmas. The results are applied to a typical model of laboratory argon plasma.

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Introduction

Plasma (charged gas) based technology underpins some of the world’s largest industries producing a significant proportion of the world’s global commercial products including computers, cell phones, automobiles, airplanes, paper, and textiles [1, 2]. Foremost among these is the electronics industry, in which plasma-based processes are indispensable for the manufacture of ultra large-scale integrated microelectronic circuits. The Boltzmann equation represents a well-defined model to describe the motion of a rarefied charged gas (plasma), when microscopic effects must be considered. This is the case of an electron flow in microelectromechanical systems (MEMS).

The choice of a physical model to adequately describe a gas flow depends on the flow regime. A rarefied gas may be divided into several different flow regimes in accordance with its level of rarefaction as quantified by the Knudsen number, Kn, defined as the ratio between the mean free path and a characteristic length scale of the gas flow. A significantly large number of flows may be classified within the transition regime (0.1 < Kn < 10). Relevant applications within a rarefied gas include upper atmospheric simulations [3], including the Space Shuttle Orbiter, the Magellan Spacecraft, the Stardust Sample Return Capsule, and the Mars Pathfinder. Additional relevance outside that of upper-atmospheric studies includes chemical vapor deposition [4], the microfilter [5], and MEMS [6–9]. Traditionally, the Boltzmann equation, based on kinetic theory, remained the only appropriate option for the solution of these rarefied gases.

The applications of the Boltzmann equation in MEMS technology are numerous. There are some important factors that increase the requirement of the employment of the Boltzmann kinetic equation in the MEMS applications as:
i. The Boltzmann equation is valid for studying the flows in all ranges of the Knudsen number (i.e., the continuum (Kn < 0.01), slip (0.01 < Kn < 0.1), transition (0.1 < Kn < 10), and free molecular regimes (10 < Kn) [10, 11]), while the Chapman–Enskog method proved the validity of the Navier–Stokes equations as a limiting case corresponding to small Knudsen numbers in the continuum regime. There were some restrictions on macroscopic equations when considering rarefied gas regimes. The attempt to construct rarefied gas dynamics similar to the macroscopic equations of hydrodynamics, without using multidimensional phase space, were unsuccessful. The need to study the Boltzmann kinetic equation itself was obvious.

ii. The Knudsen numbers of flow in MEMS are usually far from the continuum regimes because of their microscale size, which is often comparable to the molecule mean free path under standard working conditions. The kind of flow under micro-scale is called a microflow. In microflows, the characteristic lengths of the flow gradient are usually very small and comparable to the mean free path of the molecules. Typically, the characteristic lengths of most MEMS are at or below the order of a micrometre and result in a Knudsen number between 0.001 and 10, so that the flows in most MEMS are in the slip flow and transition flow regimes. Microflows in these regimes have different features than normal flows with large characteristic lengths. The conventional continuum models, unlike the Boltzmann equation, fail to describe and predict microflows. On the other hand, microflows play a role in determining the performances of MEMS, such as micropressure sensors, micropumps, and valves.

iii. The low-speed flows can often be described by the Reynolds equation, a simplified form of the Navier–Stokes equations, with negligible convective terms [9]. The Reynolds equation is often used to describe fluidic effects in microsystems with gas confined in long gaps. However, the Reynolds and Navier–Stokes description breaks down when the characteristic size decreases and the flow transitions to the rarefied regime. The Boltzmann equation is a general form of the gas transport equation based on the kinetic theory and can be reduced to Navier–Stokes equations in the near-continuum limit. For planar flows, the oscillating Couette problem has many analogies with Stokes’ second problem, which involves a flat plate oscillating in an unbounded medium. This problem was first solved by Stokes [12] in 1851 and later by Rayleigh [13] in 1911. Stokes’ second problem is one of the simplest configurations to study the behavior of a nonequilibrium gas responding to a plate oscillating in its own plane [14]. A comprehensive study has been carried out with the linearized Bhatnagar–Gross–Krook (BGK) equation over a wide range of Knudsen numbers [15]. The effect of the oscillation frequency on the amplitudes and phases of the velocity and shear stress were reported. To improve our understanding of time-periodic, shear driven gas flows, oscillatory Couette flow provides an ideal test case and has been studied extensively using kinetic theory [16–20]. A brief comparison of the linearized R13 (LR13) equations with DSMC data [18] were carried out by Taheri et al. [21]. Although oscillatory Couette flow is a simple case, it has many related applications in a variety of MEMS devices, for example, the Tang resonator [22]. A proper understanding of flow phenomena in resonators will therefore help to improve the performance and quality factor [23]. In this study, the extended continuum governing equations are employed to compute the details of Stokes’ second problem and oscillatory planar Couette flow in the early transition regime. The purpose of this paper is to analyze and assess the dynamic response capability of the higher-order moment equations for oscillatory flow and compare against the available kinetic data. Gu and Emerson [14] presented results using three different continuum-based models to study oscillatory flow in the transition regime. Data obtained from numerical solutions of the Boltzmann equation and the direct simulation Monte Carlo method, are used to assess the ability of the continuum models to capture important rarefactive effects.

The behavior of an ordinary (or rarefied) charged gas in the vicinity of an infinite flat plate oscillating or moving in its own plane — the Rayleigh flow problem — which is considered in the present study, is of great interest in laboratory experiments and aerodynamics [24]. It is important to determine the role governing the motion due to the collision of the molecules with solid surfaces and the binary collision between the molecules themselves due to the rarefaction of the gas. Therefore, discontinuities in the macroscopic parameters at the surfaces are expected. Shidlovskiy, El-Sakka et al., and Khater and El-Sharif [25–27], among other investigators, studied the Rayleigh flow problem for a highly rarefied gas of a homogeneous system of charged particles in the framework of the kinetic theory of gases using the collisionless Boltzmann equation, and they examined the dynamical and electromagnetic behavior of the gas.

Abourabia and AbdelWahid [24], in the framework of irreversible thermodynamics, examined the characteristics of the Rayleigh flow problem of a rarified electron gas extracted from neutral atoms and proved that it obeys the entropic behavior for gas systems but with an approximate solution and inaccurate formula of the collision frequency. The enhancement and improvement of this study is done in ref. 2, where the exact solution and accurate formula of electron–electron collision frequency are successfully introduced.

Kinetic theory has contributed not only to the understanding of nonequilibrium transport phenomena in gases, but also to the development of general nonequilibrium statistical physics. It is well accepted that the Boltzmann equation [28–35] is one of the most reliable kinetic models for describing nonequilibrium phenomena in gas phase. Following its success and usefulness, the Boltzmann equation is widely used to describe various gas-phase transport phenomena, such as plasma gases, granular gases, polyatomic gases, relativistic gases, and chemically reacting gases [36–39]. The kinetic equation of gas flow based on the Boltzmann equation, has obvious peculiarities in comparison with the macroscopic description found using the Navier–Stokes equations [24, 28]. Because the full Maxwell–Boltzmann equation requires more effort to solve, various approximations have been suggested [40], such as the Chapman–Enskog procedure, Krook's
model, and Lee’s moment method [41–45] for the solution of Boltzmann’s equation. Lees et al. [44] applied the two-sided Maxwellian distribution to the problem of a conductive heat transfer. Khater and El-Sharif [45] used a two-stream Maxwellian distribution function with two unknown parameters corresponding to the mean velocity and the shear stress to obtain an approximate analytical solution of the Rayleigh flow problem for a rarefied gas of an inhomogeneous system of charged particles.

The goals of this study are the following. (i) Obtaining an analytical exact solution of the unsteady Boltzmann equation of a rarefied electron gas filling the upper half of the space. The rarefied electron gas is bounded by an infinite flat plate that suddenly oscillates on its own plane, producing self-generated magnetic and electric fields. (ii) Illustrating the effects of the collisions of electrons–electrons on the form of the distribution function. In addition, (iii) indicating the nonequilibrium thermodynamic behavior of the system. For this propose, we solve the initial-boundary value problem of the Rayleigh flow problem applied to the system of the charged gas (electrons) to determine macroscopic parameters, such as the mean velocity, shear stress, viscosity coefficient, together with the induced electric and magnetic fields. Using the estimated distribution functions, it is of fundamental physical importance to study the irreversible thermodynamic behavior of the diamagnetic and paramagnetic electron gas, so that the predictions of the entropic behavior and related thermodynamic functions are investigated. The results are applied to a typical model of laboratory argon plasma.

2. The physical problem and mathematical formulation

Consider a rarefied gas of charged particles of electrons and ions occupying a semi-infinite space \((y \geq 0)\). An infinite flat plate fixed at \((y = 0)\), and parallel to the \(xz\)-plane oscillates harmonically in the \(x\)-direction with frequency \(\omega\), that is, the velocity of the plate depends on the time \(t\) as

\[
V_w = R_c[U_0 \exp(-i\omega t)] = U_0 \cos(\omega t)
\]

where the symbol \(R_c\) denotes the real part of complex expression. The quantity \(U_0\) is the velocity amplitude, which is assumed to be small when compared with the thermal molecular velocity \(V_T\) of the gas. Because the ratio between electron and ion masses and electron and neutral atom mass is small \((m_e/m_i \ll 1, m_e/m_n \ll 1)\) in ionized gases, the ions and neutrals will be considered an immobile neutralizing background. The charged gas is initially in absolute equilibrium and the wall is at rest. Then the plate starts to oscillate suddenly in its own plane with a velocity \((U_0 \cos(\omega t))\) along the \(x\)-axis \((U_0\) and \(\omega\) are constants). Moreover, the plate is considered impermeable, uncharged, and an insulator. The whole system (electrons + ions + plate) is kept at a constant temperature. All physical quantities are defined in the nomenclature.

Let the force \(f_e\) acting on each electron; be given by [46–49]

\[
f_e = -eE - \frac{e}{c_0}(e \times B)
\]

By taking

\[
V = (V_x, 0, 0) \quad J = (qnV_x, 0, 0) \quad E = (E_x, 0, 0)
\]

and \(B = (0, 0, B_z)\)

We assume that \(V_x, E_x, B_z\), and \(J_x\) are functions of \(y\) and \(t\).

This choice satisfies Maxwell’s equations. The distribution function \(F\) of the particles for the electron gas can be obtained from the kinetic Boltzmann’s equation, which can be written in the BGK model [50] as

\[
\frac{\partial F_e}{\partial t} + c \cdot \frac{\partial F_e}{\partial r} + \frac{f_e}{m_e} \cdot \frac{\partial F_e}{\partial c_e} = \nu_{ee}(F_{0e} - F_e)
\]

where

\[
F_{0e} = n(2\pi RT)^{-3/2} \exp\left(-\frac{(c - V)^2}{2RT}\right)
\]

The quantities \(n, V, T\) are the number density, mean drift velocity, and effective temperature obtained by taking moments of \(F_e\).

In addition, we assume that the particles are reflected from the plate with a full velocity accommodation, that is, the charged gas particles are reflected with the plate velocity so that the boundary conditions are taken as:

\[
V_x(0, t) = U_0 \cos(\omega t) \quad \text{for} \quad t > 0
\]

where \(V_x = V_x\) as \(c_y > 0\) and \(V_x\) is finite as \(y \to \infty\).

Substituting from (1) and (2), into (3) one obtains

\[
\frac{\partial F_e}{\partial t} + c_y \frac{\partial F_e}{\partial y} - \frac{eB_z}{m_e} \left( \frac{c_x \frac{\partial F_e}{\partial c_x} - c_y \frac{\partial F_e}{\partial c_y}}{m_e} \right) + \frac{eE_x}{m_e} \frac{\partial F_e}{\partial c_x} = \nu_{ee}(F_{0e} - F_e)
\]

where \(\nu_{ee}\) is the electron–electron collision frequency, which is given by [51–54]

\[
\nu_{ee} = \frac{4\sqrt{\pi ne^4 \log[A_{ee}]} }{3 \sqrt{m_e k_B^2 T^{3/2}}}
\]

where \(\log[A_{ee}] = \log[4\pi n Z^2 \lambda_{De}^3]\), \(Z\), and \(\lambda_{De}\) are the Coulomb logarithms, the degree of ionization, and Debye radius, respectively.

The moment method [55] for the solution of Boltzmann’s equation is employed here. One of the most important advantages of this method is that the surface boundary conditions are easily satisfied. Maxwell converted the Maxwell–Boltzmann equation into an integral equation of transfer, or moment equation, for any quantity \(Q(e)\) that is a function only of the molecular velocity. The distribution function used there should be considered a suitable weighting function, which is not the exact solution of the Maxwell–Boltzmann equation in general. Lees found that the distribution function employed in Maxwell’s moment equation must satisfy the following basic requirements: (i) it must have the “two-sided” characteristic that is an essential feature of highly rarefied gas flows; (ii) it must be capable of providing a smooth transition from free molecule flows to the continuum regime; and (iii) it should lead to the simplest possible set of differential equations and boundary conditions consistent with conditions \(i\) and \(ii\).
Let us write the solution of (4), in the form [25]

\[
F_e = \begin{cases} 
F_1 = n(2\pi RT)^{-3/2}(1 + c_y V_{s1}) \exp \left( \frac{-c^2}{2RT} \right) & \text{for } c_y < 0 \\
F_2 = n(2\pi RT)^{-3/2}(1 + c_y V_{s2}) \exp \left( \frac{-c^2}{2RT} \right) & \text{for } c_y > 0
\end{cases}
\]

where \( V_{s1} \) and \( V_{s2} \) are two unknown functions of time \( t \) and the single distance variable \( y \).

The solution of the Boltzmann equation is difficult, and the velocity distribution function \( F \) is not directly of interest to us at this stage, but the moments of the distribution function are of interest. Therefore, we derive Maxwell’s moment equations by multiplying the Boltzmann equation by a function of velocity \( Q_j(e) \) and integrating over the velocity space. How many and what forms of \( Q_j(e) \) will be used is dependent on how many unknown variables need to be determined and how many equations need to be solved.

Using Grad’s moment method [41] multiplying (4) by \( Q_j(e) \) and integrating over all values of \( e \), we obtain the transfer equations in the form

\[
\frac{\partial}{\partial t} \int Q_j F e d\xi + \frac{\partial}{\partial y} \int e Q_j \frac{\partial F e}{\partial y} d\xi + \frac{e E_e}{m_e} \int F e \frac{\partial Q_j}{\partial e} d\xi - \frac{e B_e}{m_e c_0} \int \left( c_x \frac{\partial Q_j}{\partial c_y} - c_y \frac{\partial Q_j}{\partial c_x} \right) d\xi = v_{ee} \int (F_{0e} - F_e) Q_j d\xi
\]  

(7)

The integrals over the velocity distance are evaluated from the relation [25]

\[
\int Q_j F e d\xi = \int \int \int \int Q_j F_1 e d\xi + \int \int \int Q_j F_2 e d\xi
\]

(8)

where \( Q_j = Q_j(e) \), \( j = 1, 2 \), and \( d\xi = dc_x dc_y dc_z \), where \( c_x \), \( c_y \), and \( c_z \) are the particles’ velocity components along \( x \)-, \( y \)-, and \( z \)-axes, respectively. Moreover, \( E_e \) and \( B_e \) may be obtained from Maxwell’s equation

\[
\frac{\partial E_x}{\partial y} - \frac{1}{c_0} \frac{\partial B_y}{\partial t} = 0
\]  

(9)

\[
\frac{\partial B_x}{\partial y} - \frac{1}{c_0} \frac{\partial E_y}{\partial t} - \frac{4\pi e n}{c_0} V_x = 0
\]  

(10)

where

\[
n = \int F d\xi \quad nV_x = \int c_x F d\xi
\]

with the initial and boundary conditions

\[
E_x(y, 0) = B_x(y, 0) = 0 \\
E_x(y, t) \quad \text{and} \quad B_x(y, t) \quad \text{are finite as } y \to \infty
\]  

(11)

We introduce the dimensionless variables defined by

\[
t = t' \tau_{ee} \\
y = y' \left( \frac{\tau_{ee} V_{Te}}{\sqrt{2\pi}} \right) \\
V_x = V_x' V_{Te}
\]

\[
\tau_{xy} = \tau_{xy}' V_{Te} \\
Ma = \frac{U_0}{V_{Te}} \\
B_t = B_t' \left( \frac{\sqrt{2\pi m_e c_0}}{e\tau_{ee}} \right)
\]

\[
E_x = E'_x \left( \frac{m_e V_{Te}}{e\tau_{ee}} \right) \\
\rho = nm \\
V_{Te} = \sqrt{\frac{2kBT_e}{m_e}}
\]

\[
\epsilon = \frac{m_e}{m_i} \quad dU = dU'(k_BT_e) \quad \text{and}
\]

\[
F_j = F'_j n_e (2\pi RT_e)^{-3/2} \quad j = 0, 1, 2
\]  

(12)

For \( \text{Ma}^2 \ll 1 \) (low Mach number), we can assume that the density and the temperature variation at each point of the flow and at any time are negligible, that is, \( n = 1 + O(\text{Ma}^2) \) and \( T = 1 + O(\text{Ma}^2) \). Let

\[
V_x = \frac{1}{2} (V_{s1} + V_{s2})
\]

\[
\tau_{xy} = \frac{P_{xy}}{\rho U_0 \sqrt{RT_e/2\pi}} = (V_{s2} - V_{s1})
\]  

(13)

where \( P_{xy} \) is the shear stress [42] defined by

\[
P_{xy} = m \int (c_x - V_x') c_y F d\xi
\]

We take \( Q_1 = c_x \) and \( Q_2 = c_x c_y \), and substitute (6) into (7), using the dimensionless variable defined in (12). After dropping the bars, we get the following equations (neglecting the displacement current) [56]:

\[
\frac{\partial V_x}{\partial t} + \frac{\partial \tau_{xy}}{\partial y} - E_x = 0
\]  

(14)

\[
\frac{\partial \tau_{xy}}{\partial t} + 2\pi \frac{\partial V_x}{\partial y} + \tau_{xy} = 0
\]  

(15)

\[
\frac{\partial E_x}{\partial y} - \frac{\partial B_y}{\partial t} = 0
\]  

(16)

\[
\frac{\partial B_x}{\partial y} - \alpha_0 V_x = 0
\]  

(17)

where
With the initial and boundary conditions
\[ V_x(y, 0) = \tau_{xy}(y, 0) = E_x(y, 0) = B_z(y, 0) = 0 \]
\[ 2V_x(0, t) + \tau_{xy}(0, t) = 2Ma \cos(\beta_1 t) \quad \text{for} \quad t > 0 \]
\[ V_x, \tau_{xy}, E_x, \text{and } B_z \text{ are finite as } y \to \infty \]

We can reduce basic equations (14)–(17), after simple algebraic manipulations to a single equation
\[ \alpha_0 = \frac{V_{\text{Te}}^2 r_{\text{ee}} c^2 n}{m_e c_0^2} \]

With the initial and boundary conditions
\[ V_x(y, 0) = \tau_{xy}(y, 0) = E_x(y, 0) = B_z(y, 0) = 0 \]
\[ 2V_x(0, t) + \tau_{xy}(0, t) = 2Ma \cos(\beta_1 t) \quad \text{for} \quad t > 0 \]
\[ V_x, \tau_{xy}, E_x, \text{and } B_z \text{ are finite as } y \to \infty \]

We can reduce basic equations (14)–(17), after simple algebraic manipulations to a single equation
\[ \alpha_0 \frac{\partial^4 V_x(y, t)}{\partial t^2 \partial y^2} - \frac{2\pi}{\partial^4 V_x(y, t)} - \frac{\partial^4 V_x(y, t)}{\partial t^2 \partial y^2} = -\alpha_0 \frac{\partial^2 V_x(y, t)}{\partial t^2} - \alpha_0 \frac{\partial V_x(y, t)}{\partial t} = 0 \]

### 3. Solution of the initial-boundary value problem

We will use the traveling wave solution method [57, 58] considering
\[ \xi = ly - mt \]

such that all the dependent variables are functions of \( \xi \). Here \( l \) and \( m \) are transformation constants that do not depend on the properties of the fluid but are parameters to be determined by the boundary and initial conditions [58]. From (20) we get the derivatives
\[ \frac{\partial}{\partial t} = -m \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} = l \frac{\partial}{\partial \xi} \]
\[ \frac{\partial^a}{\partial \xi^a} = (-1)^a m^a \frac{\partial^a}{\partial \xi} \]
\[ \frac{\partial^a}{\partial \xi^a} = l^a \frac{\partial^a}{\partial \xi^a} \]

where \( a \) is a positive integer.
Substituting from (20) and (21) into (19) to get
\[
\frac{m^2 l^2}{C_0^2} p m^4 \frac{V_x(x)}{C_0 / C_1} d^4 x + \frac{m l^2}{C_0} d^3 V_x(x) d^2 x + \frac{a_0 m}{C_0} d^2 V_x(x) dx + \frac{a_0 m}{C_0} d V_x(x) dx = 0
\] (22)

The boundary and initial conditions become
\[
E_x(x = 0) = B_z(x = 0) = \tau_{xy}(x = 0) = 0
\]
\[
2 V_x(x = -m) + \tau_{xy}(x = -m) = 2 M_p \cos(\beta_1)
\]
at \( y = 0 \) e.g., \( t = 1 \)
\[
V_x, \tau_{xy}, E_x, \text{ and } B_z \text{ are finite as } x = -\infty
\] (23)

Now, we have an ordinary differential (22) with the boundary and initial conditions (23).

The ordinary fourth-order homogeneous differential (22), can be successfully solved exactly, with the help of symbolic computer software, with their boundary and initial conditions (23). The sought solutions will be applied to a typical model of laboratory argon plasma.

4. The investigation of the behavior of the internal energy change

The studying of the behavior of the internal energy change for the physical systems, presents a great importance in science. To study the internal energy change for the system,
based on the solution of the nonstationary Boltzmann equation [59], we introduce the extended Gibbs relation [2, 24, 60]. It includes the electromagnetic field energy as a part of the whole energy balance, which distinguishes the charged gas into paramagnetic and diamagnetic ones. If there are unpaired electrons in the molecular orbital diagram, the gas is paramagnetic, and if all electrons are paired, the gas is considered a diamagnetic one. To get the work term in the first law of thermodynamics, we should write the internal energy balance including the electromagnetic field energy as follows.

4.1 Paramagnetic plasma

The internal energy change is expressed in terms of the extensive quantities $S$, $P$, and $M$, which are the thermodynamic coordinates corresponding to the conjugate intensive quantities $T$, $E$, and $B$, respectively. The three contributions in the internal energy change in the Gibbs formula

$$dU = dU_S + dU_{\text{pol}} + dU_{\text{par}}$$

where $dU_S = TdS$ is the internal energy change due to variation of the entropy $S$, where the entropy per unit mass $S$ [15, 23, 52]

$$S = -\pi^{V^2} \left( \frac{V^2}{x_1} + \frac{V^2}{x_2} \right) - \frac{3}{2}$$

where $dU_{\text{pol}} = EdP$ is the internal energy change due to variation of polarization $P$, and $dU_{\text{par}} = BdM$ is the internal energy change due to the variation of magnetization, here $M$ is calculated from the equation [24, 60].

Fig. 5. The induced electric field $E_x$ versus space $y$ and time $t$.

Fig. 6. The induced magnetic field $B_z$ versus space $y$ and time $t$. 
Introducing the dimensionless variables

\[ U_0 = \frac{U}{K T}, \quad M_0 = \frac{M}{e \tau_{ee} V_{Te}}, \quad P_0 = \frac{P}{e \tau_{ee} V_{Te}} \]

in the Gibbs formula to get (after dropping the primes)

\[ dU = dS + f_1 E dP + f_1 B dM \]  

(27)

4.2 Diamagnetic plasma

On the other hand, if the plasma is diamagnetic; the internal energy change due to the extensive variables \( S, P, \) and \( B \) represent the thermodynamic coordinates conjugate to the intensive quantities \( T, E, \) and \( M, \) respectively, therefore, we have three contributions in the internal energy change in the Gibbs formula given by

\[ dU = dU_S + dU_{pol} + dU_{dia} \]  

(28)
where $dU_{\text{dia}} = -Md_b$ is the internal energy change due to the variation of the induced magnetic induction, where $M = T(\partial S/\partial B)$ [24, 60].

Hence, the dimensionless form for $dU$ in this case takes the form

$$dU = dS + f_1 E dP - f_1 M dB$$  \hspace{1cm} (29)

where

$$f_1 = \left( \frac{m_e V_{\text{Th}}^2}{K T} \right), \quad dS = \left( \frac{\partial S}{\partial r} \right) \delta y + \left( \frac{\partial S}{\partial t} \right) \delta t, \quad \delta y = 5$$  \hspace{1cm} \delta t = 5

5. Discussion

In this problem, the unsteady behavior of a rarefied electron gas is studied based on the kinetic theory of irreversible processes using the exact traveling wave analytical solution method via the BGK model of the Boltzmann equation with the exact value of electron–electron collision frequency. Our computations are performed according to typical data for electron gas in argon plasma [61] as a paramagnetic medium in the case where the argon gas loses single electrons or as a diamagnetic medium in the case where the argon gas loses electron pairs, depending on the ionizing potential applied to the argon atoms. The following conditions and parameters
are applied: $k_B = 1.3807 \times 10^{-22}$ J/K, $T_0 = 1200$ K, $n_e = 7 \times 10^{13}$ cm$^{-3}$, $d = 3.84 \times 10^{-8}$ cm (diameter of the argon atom), the electron rest mass and charge $m_e = 9.093 \times 10^{-28}$ g, $e = 4.8 \times 10^{-10}$ esu (1 esu = 0.333 564 1 nC) are used to calculate the dimensionless parameter $\alpha_0 = 1.22 \times 10^{-6}$, the electron-electron collision relaxation time $\tau_{ee} = 1.306 \times 10^{-11}$, and the mean free path of the electron gas

$$\lambda = \frac{1}{2\pi n_e d^2} = 2.180\text{ cm}$$

compared to the electron Debye length

$$\lambda_{De} = \sqrt{\frac{K_0 T_0}{4\pi n_e e^2}} = 2.85 \times 10^{-5}\text{ cm} \quad f_1 = 2$$

Using the idea of the shooting numerical calculation method, we evaluate the transformation constants to obtain $m = 1.3, \beta = 0.1$, and the plate Mach number $Ma = 3.3 \times 10^{-2}$.

All the variable of the problem satisfies the initial and the boundary conditions, (18), of the problem (see Figs. 2, 3, 5, and 6).

Figure 1 shows that the deviation from equilibrium is small and over the course of time the perturbed velocity distribution functions $F_1$ and $F_2$ approach equilibrium velocity distribution function $F_{0e}$. This is in good agreement with the famous Le Chatelier principle. Figure 2 illustrated that the mean velocity of electrons, near the oscillating plate, has a maximum value equal to the Mach number ($= Ma$) of the plate, which satisfies the condition of the problem. The negative sign of the shear stress represents its direction, from down to up, and the shear stress decreases with time due to the behavior of the velocity itself, see Fig. 3.

The viscosity coefficient [62] for a rarefied gas in a slow flow is $\mu = \tau_{eff}(\partial V_s/\partial y)$. Its behavior is represented in Fig. 4, which shed light upon the resistance to the motion, which gradually increases with time. This is due to the fact that, if any change is imposed on a system that is in equilibrium, then the system tends to adjust to a new equilibrium countering the change. This gives an agreement with the Le Chatelier principle. Near the oscillating plate, Figs. 5 and 6 indicate the behavior of the self-generated fields. The induced electric field increases with time while the induced magnetic field decreases. Clearly, they satisfy the conditions of the problem.

Upon passing through a plasma, a charged particle (electron) loses (or gains) part of its energy because of the interaction with the surroundings because of plasma polarization and collisions [63]. The energy loss (or gain) of an electron is determined by the work of the forces acting on the electrons in the plasma, by the electromagnetic field generated by the moving particle itself (because the suddenly oscillating plate causes work to be done on the gas), and this produces a change in the internal energy of the gas $U$. As seen in Figs. 7–10, the change in the internal energy due to the variation of entropy and paramagnetism is smoothly dampened with time by energy lost to and gained from the ions and plate, respectively. While the change in internal energy increases gradually with time because of the intensive variables, corresponding to either polarization or diamagnetic plasma.

6. Conclusion

The solution of the unsteady BGK Boltzmann kinetic equation in the case of a rarefied electron gas using the method of the moments of the two-sided distribution function together with Maxwell’s equations is developed within the traveling wave exact solution method and the exact value of electron–electron collision frequency. Tackling this allows one to calculate the components of the flow velocity. Inserting the velocity components into the suggested two-sided distribution functions and applying the Boltzmann H-theorem, we can evaluate the entropy, entropy production, thermodynamic force, and kinetic coefficient. Via Gibbs’ equations, the ratios between the different contributions of the internal energy change are evaluated based upon the total derivatives of the extensive parameters. The predictions, estimated using Gibbs’ equations, reveal the following order of maximum numerical magnitude ratios between the different contributions to the internal energy change, based on the total derivatives of the extensive parameters:

$$dU_S : dU_{pol} : dU_{dia} = 1 : 2.5 \times 10^{-1} : 3.1 \times 10^{-4}$$

$$dU_S : dU_{pol} : dU_{par} = 1 : 2.5 \times 10^{-1} : 3.7 \times 10^{-4}$$

It is concluded that the effect of the changes in the internal energies $dU_{pol}, dU_{dia}$ and $dU_{par}$ due to electric and magnetic fields are very small in comparison with $dU_S$, in recognition of the fact that these fields are self-induced by the sudden motion of the plate.

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References

\( dU_{\text{par}} \) internal energy change due to the variation of magnetization

\( dU_{\text{pol}} \) internal energy change due to the variation of polarization

\( dU_{S} \) internal energy change due to the variation of entropy

\( e \) electron charge

\( E \) induced electric vector

\( E_{x} \) induced electric field in the \( x \)-direction

\( E_{x}' \) dimensionless induced electric field in the \( x \)-direction

\( F \) velocity distribution function

\( F_{\text{0e}} \) local maxwellian distribution function for electrons

\( F_{1} \) distribution function for descending particles \( c_{y} < 0 \)

\( F_{2} \) distribution function for ascending particles \( c_{y} > 0 \)

\( F_{e} \) velocity distribution function for electrons

\( F_{j} \) velocity distribution function

\( F_{j}' \) dimensionless velocity distribution function

\( f_{e} \) Lorentz’s force vector

\( J \) current density vector

\( J_{x} \) current density in the \( x \)-direction

\( K_{B} \) Boltzmann constant \( 1.3807 \times 10^{-22} \) J/K

\( \text{Kn} \) Knudsen number

\( M \) specific magnetization

\( M' \) dimensionless specific magnetization

\( M_{a} \) plate Mach number

\( m_{e} \) electron mass

\( m_{i} \) ion mass

\( m_{N} \) neutral mass

\( n \) mean density

\( n_{e} \) electron concentration

\( P \) polarization

\( P' \) dimensionless polarization

\( P_{xy} \) shear stress

\( Q(c) \) a function only of the molecular velocity

\( q \) charge

\( R \) gas constant

\( R_{e} \) gas constant for electrons

\( r \) position vector of the particle

\( S \) entropy per unit mass

\( T \) temperature

\( T_{e} \) electron temperature

\( t \) time variable

\( t' \) dimensionless time variable

\( U' \) dimensionless internal energy of the gas

\( U_{0} \) Plate initial Velocity

\( V_{fe} \) electron thermal velocity

\( V_{s} \) mean velocity

\( V_{x} \) Dimensionless mean velocity

\( V_{s1} \) mean velocity related to \( F_{1} \)

\( V_{s2} \) mean velocity related to \( F_{2} \)

\( y \) displacement variable

\( y' \) dimensionless displacement variable

\( Z \) degree of ionization

\( \alpha \) frequency

\( \alpha_{0} \) dimensionless parameter

\( \beta_{1} \) dimensionless frequency

\( \varepsilon \) mass ratio

\( A_{\text{De}} \) Coulomb logarithm

\( \lambda \) mean free path

\( \lambda_{\text{De}} \) Debye radius of electrons

\( \mu \) viscosity coefficient

\( v \) electron–electron collision frequency

\( \xi \) wave parameter

\( \rho \) density

\( \tau \) relaxation time

\( \tau_{ee} \) electron–electron relaxation time

\( \tau_{xy} \) shear stress

\( \tau_{xy}' \) dimensionless shear stress